Complex Variables Basic Course

List of topics

Complex Numbers

1. The fields of complex numbers, geometrical interpretation of arithmetical operations, Moivre's formula.
2. Basic topology in the complex plane: compact spaces, connected spaces, stereographic projection.
3. Successions and complex series, convergence criteria, (comparison, Abel, Weierstrass M-test)
4. Potency series, disk of convergence, Cauchy-Hadamard formula, specific series for elementary functions.
5. Elementary conformal transformations, Möbius transformations, subgroups that preserve disk or semi-planar, cross reasons, symmetry.

Holomorphic Functions

1. Cauchy-Riemann equations, Harmonic functions and harmonic conjugates, Goursat's theorem.
2. Conform properties of holomorphic functions
3. Analyticity of holomorphic functions, differentiation of potency series

III. Curves and Integration

1. Linear integrals (ds, dz, |dz|), curve length, homotopy between curves
2. Cauchy Theorem and Integral, launching index
3. Local primitive of a holomorphic or harmonic function
4. Consequences of Cauchy integral: Morera and Liouville theorems, foundations of algebra, Maximum principle and Schwarz lemma.

Singularities

1. Zeros, poles and elementary singularities. Riemann theorem on removable singularities, Casorati-Weierstrass theorem
2. Laurent series
3. Calculation of residuals: Residue theorem and applications. Argument principle. Rouche theorem. Calculation of real definite integrals.
4. Rational functions as meromorphic functions on S^2, order of a rational function, partial fraction decomposition.

Reference

Ahlfors, L.V.	Complex Analysis
Knopp, K.	Elements of the Theory of Functions I, II
Markushevich, A.I.	Theory of Functions of a Complex Variable I, II
Cartan, H.	Theory of Analytic Functions
Conway, J.	Functions of One Complex Variable
Beardon, A.F.	Complex Analysis: The Argument Principle in Analysis and Topology
Grove, E.A., Ladas, G.	Introduction to Complex Variables
Silverman, R.	Introductory Complex Analysis