

# ***Topology Basic Course***

## ***List of Topics***

### **I. Elementary Concepts**

1. Topological spaces. Basis and fundamental systems of neighborhoods
2. Interior, lock and frontier. Complementation.
3. Continuity. Initial and final topology. Subspace, quotient, sum and product topologies.
4. Compact spaces. Tychonoff theorem, Local properties.
5. Connected spaces. Connected spaces by trajectories. Local properties.
6. Separable spaces and second-countable space topologies. Succession convergence.
7. Urysohn lemma and Tietze theorem
8. Space compaction. Metric theorem.
9. Examples: Euclidean topology, domain invariance, Metric spaces, topological groups (general linear groups, orthogonal and unit groups, Gram-Schmidt orthogonalization process, varieties (spheres, projective spaces, surfaces)

### **II. Functions and Homotopy Spaces**

1. Function spaces. Compact-open and point convergence topologies
2. Adjunction and natural topology. Continuity of composition and evaluation.
3. Stone-Weierstrass and Ascoli theorems. Baire spaces.
4. Homotopies between curves and functions. Fundamental group.
5. Cones and suspensions. Cone extension
6. Loop spaces. Homotopy groups.

### **III. Fibered loops**

1. Locally trivial beams
2. Paracompact spaces. Unit partitioning
3. Lifting of functions and homotopy on fibered beams
4. Vectorial beams. Example: tangent beam on a variety
5. Stiefel and Grassman varieties. Universal beams
6. Covering spaces. Lifting of curves and functions.
7. Classification of covering spaces. Universal cover. Fundamental group of the circle
8. Applications: Tangent fields and fixed points, Jordan separation theorem, Algebra fundamental theorem, classification of topological groups. Brouwer fixed point theorem in two dimensions.

### **IV. Cellular complexes**

1. Quotient topology and adjoint spaces
2. Cellular complexes and paracompact spaces
3. Cell decomposition of spheres and projective spaces
4. Hopf fibrations  $S^{2n-1} \rightarrow S^n$  (unique cases:  $n=1,2$  and  $8$ )

5. Cell decomposition of Stiefel and Grassmann varieties.
6. Extension of functions (cf. Tietze extension theorem)
7. Homological curves and homology first group of space
8. Poincaré-Hurewicz theorem

## Reference

Adams, J.F.	Algebraic Topology: A Students Guide
Atiyah, M.F.	K--Theory
Bourbaki, N.	General Topology
Dugundji, J.	Topology