New Trends in Analysis and Mathematical Physics

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CINVESTAV Departamento de Matemáticas Campus Querétaro

Jeanneth Galeano
(Departamento de Matemáticas, Cinvestav Campus Querétaro)
Regularization Effect in Certain Parabolic Equations

In this talk we introduce the space of infinitely pseudo-differentiable functions with respect to the operator defined as $\mathbf{f}(D;\alpha)\phi = \mathcal{F}^{-1}(\,|f(\xi)|_p^{\alpha}\,\mathcal{F}(\phi)\,)$, where $\alpha > 0$, $\mathcal{F}$ denotes the Fourier transform, ϕ is a function of Lizorkin type and f is a quasielliptic polynomial. By using these spaces we show the existence of a regularization effect for certain parabolic equations over p-adics.

Hugo García Compeán
(Depto. de Física, Cinvestav)
The Problem of Motion in String Theory

We survey the description of motion in some physical theories of interest. A very important difference is pointed out between linear theories like Newtonian gravity and Electrodynamics and non-linear ones such as Yang-Mills and General Relativity. After that we argue on the problem of motion in string theory. In particular, after setting the problem in this context we describe this problem in supergravity coming from string theory and in Witten open string field theory.

Tim Gendron
(IMATE-U.N.A.M. Cuernavaca)
Nonstandard Geometry and the Quantum Modular Invariant

The purpose of the talk is to propose nonstandard geometry as an alternative to noncommutative geometry for studying quantum spaces. The main point is that nonstandard geometry, in contrast to noncommutative geometry, has a good notion of fundamental group.

To illustrate this principle, consider the quantum torus $T(r)$ associated to a real irrational r. We will show that the group ${}^*\mathbb{Z}(r)$ of Diophantine approximations of r, a subgroup of the nonstandard integers, plays the role of fundamental group for $T(r)$. Then we use ${}^*\mathbb{Z}(r)$ to define the quantum modular invariant ${}^*j(r)$ of $T(r)$, which by definition is a nonstandard real number. Conjecturally, ${}^*j(r)$ is asymptotic to a standard real number; we will show this for an infinite family of quadratic irrationals. This could be applied to solve Yu. Manin's Real Multiplication Program.

Benjamín Itzá
(CIMA–Pachuca)
Schwartzman cone in the K-theory of C*-dynamical systems

In 1957 Sol Schwartzman made one of the first applications of algebraic topology to topological dynamics by associating to each continuous flow on $(Y,\mathbb{R})$ with $\mathbb{R}$-invariant measure ν, an "average asymptotic cycle" $A_\nu\colon H^1(Y,\mathbb{Z}) \to \mathbb{R}$. Schawartzman also showed that the eigenvalues of the flow are contained in the range of A_ν. Assuming that $(Y,\mathbb{R})$ admits a cross section $(X,\mathbb{Z})$, in this talk we show how to compare A_ν with a trace τ of the K_0 group of the cross product C^*-algebra associated to $(X,\mathbb{Z})$.

Anatoly Kochubei
(National Academy of Sciences of Ukraine)
Non-Archimedean Analysis. An Introduction

In an introductory talk we explain, in elementary form, the main notions of non-Archimedean analysis and describe the most promising research directions in this field.

Anatoly Kochubei
(National Academy of Sciences of Ukraine)
Non-Archimedean Spectral Theorems

We describe some classes of bounded linear operators on Banach spaces over non-Archimedean fields admitting orthogonal, in the non-Archimedean sense, spectral decompositions. The main result, new even for finite matrices, is a direct analog of the classical spectral theorem but is based on different structures, due to the absence in the non-Archimedean situation of a nontrivial involution and an inner product agreed with the norm. As a special case, we consider the counterparts of unitary operators and present an analog of Stone's theorem on representation of one-parameter unitary groups.

Anatoly Kochubei
(National Academy of Sciences of Ukraine)
Fractional Diffusion Equations

Evolution equations with the Caputo-Dzhrbashyan fractional derivatives and their generalizations are widely used in physics of disordered systems for the description of anomalous diffusion in media of fractal nature, for which the average square displacement of a diffusive particle grows, for large time, slower than usual. The mathematical theory of such equations is a modern extension of the classical theory of parabolic partial differential equations. Our survey talk will include fractional analogs of the heat equation, fractional-parabolic systems, equations with the distributed order derivatives and their generalizations.

Ernesto Lupercio
(Depto. de Matemáticas, Cinvestav Campus Zacatenco)
Non-commutative tropical geometry

In this talk I will explain our approach to non-commutative tropical geometry, which generalizes ordinary tropical geometry much as non-commutative geometry generalizes ordinary algebraic geometry. Various parts of this work are joint with Katzarkov, Meersseman, Mikhalkin, and Verjovsky.

Jacob Mostovoy
(Depto. de Matemáticas, Cinvestav Campus Zacatenco)
The space of Snyder-Yang-Leznov

The space of Snyder-Yang-Leznov is a 3-parametric commutative deformation of the Poisson algebra of a particle in 3-space. We shall review some integrable systems in this space and discuss its physical relevance.

Robert Oeckl
(IMATE–U.N.A.M., Morelia)
Holomorphic quantization in background-independent quantum field theory

Most quantization schemes are designed to generate as output a Hilbert space of states and an algebra of operators acting on it, corresponding in some way to classical observables. These structures are usually thought of as defining a quantum theory in the sense of yielding coherent rules for the (probabilistic) prediction of physical measurement outcomes. This is the case, however, only in the presence of a fixed background metric which is an essential ingredient of these rules. In the absence of a fixed background metric one way to define a quantum theory consistently is via the general boundary formulation. In this formulation of quantum theory, a Hilbert space of states is associated to each hypersurface in spacetime. Moreover, for any region of spacetime, an amplitude is associated to each state on its boundary. A quantization scheme must thus produce these Hilbert spaces and amplitudes from a classical theory. After a brief review of the general boundary formulation, we present a quantization scheme with this property for linear field theories. The quantization scheme represents a synthesis of ingredients from geometric quantization on the one hand and the Feynman path integral on the other. It is mathematically rigorous and might be viewed as a functor from a category of classical field theories to a category of quantum field theories.

John Jaime Rodríguez
(Universidad Nacional de Colombia)
Some Pseudo-differential Equations Associated with the Bessel Potential

In this short talk we study some pseudo-differential equations associated with the Bessel potential

$$\mathcal{J}^{\alpha}u = \mathcal{F}^{-1}(\max(1, |\,\xi|)^{-\alpha}\mathcal{F}(u)), \quad u \in S.$$

We study the equation $\mathcal{J}^{\alpha}u = g$, and the evolution equation

$$\frac{\partial u(x,t)}{\partial t} + a\mathcal{J}^{\alpha}u(x,t) = 0, \quad x \in \mathbb{Q}_p, \quad t > 0.$$

The results presented are part of work in progress.